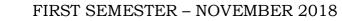
LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS



MT 1500 – ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY

 Date: 24-10-2018
 Dept. No.
 Max. : 100 Marks

 Time: 09:00-12:00
 Max. : 100 Marks

PART – A

Answer ALL questions

- 1. Find the nth derivative of $y = \log(ax + b)$.
- 2. Find the polar subtangent and polar subnormal of the curve $r = a\theta$.
- 3. Write the Cartesian formula for the radius of curvature.
- 4. Define evolute of a curve.
- 5. Find the equation, with rational coefficients one of whose roots is $\sqrt{5} + \sqrt{2}$
- 6. Calculate the sum of the cubes of the roots of the equation $x^4 + 2x + 3 = 0$.
- 7. Show that $\cosh^2 x \sinh^2 x = 1$.
- 8. Write the expansion of $\sin n\theta$.
- 9. Define conjugate diameter of an ellipse.
- 10. Find the asymptotes of the hyperbola $3x^2 5xy 2y^2 + 17x + y + 14 = 0$.

PART - B

Answer any FIVE questions

 $(5 \times 8 = 40)$

 $(10 \times 2 = 20)$

- 11. Find the nth differential coefficient of $\cos x \cdot \cos 2x \cdot \cos 3x$.
- 12. Using Lagrange's multipliers method find the minimum value of u, where $u = a^3x^2 + b^3y^2 + c^3z^2$ with the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.
- 13. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$ and $y = a(1 \cos \theta)$ is $4 \cos \frac{\theta}{2}$.
- 14. Solve the equation $81x^3 18x^2 36x + 8 = 0$ whose roots are in harmonic progression.
- 15. Express $\frac{\sin 6\theta}{\sin \theta}$ in terms of $\cos \theta$.
- 16. Show that the eccentric angles at the extremities of a pair of semi conjugate diameters of an ellipse differ by a right angle.

- 17. Derive the polar equation $\frac{l}{r} = 1 + e \cos \theta$ of a conic.
- 18. Show that in a conic the semi-latus rectum is the harmonic mean between the segments of a focal chord.

PART - C

 $(2 \times 20 = 40)$

Answer any TWO question

- 19. If $y = \sin(m \sin^{-1} x)$, prove that $(1 x^2)y_2 xy_1 + m^2 y = 0$ and $(1 x^2)y_{n+2} (2n+1)xy_{n+1} + (m^2 n^2)y_n = 0$.
- 20. (a) Find the angle of intersection of the curves $r = a(1 + \cos \theta)$ and $r = b(1 \cos \theta)$.
 - (b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
- 21. Solve the equation $6x^6 35x^5 + 56x^4 56x^2 + 35x 6 = 0$.
- 22. (a) Sum to infinity $c \sin \alpha \frac{c^2}{2} \sin 2\alpha + \frac{c^3}{3} \sin 3\alpha + \cdots \infty$.

(b) If e and e_1 are two extremities of hyperbola and its conjugate, show that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$.
