



**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2018

**MT 1500 – ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY**

Date: 24-10-2018

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

**PART – A**

**Answer ALL questions**

**(10 x 2 = 20)**

1. Find the  $n^{\text{th}}$  derivative of  $y = \log(ax + b)$ .
2. Find the polar subtangent and polar subnormal of the curve  $r = a\theta$ .
3. Write the Cartesian formula for the radius of curvature.
4. Define evolute of a curve.
5. Find the equation, with rational coefficients one of whose roots is  $\sqrt{5} + \sqrt{2}$
6. Calculate the sum of the cubes of the roots of the equation  $x^4 + 2x + 3 = 0$ .
7. Show that  $\cosh^2 x - \sinh^2 x = 1$ .
8. Write the expansion of  $\sin n\theta$ .
9. Define conjugate diameter of an ellipse.
10. Find the asymptotes of the hyperbola  $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$ .

**PART – B**

**Answer any FIVE questions**

**(5 x 8 = 40)**

11. Find the  $n^{\text{th}}$  differential coefficient of  $\cos x \cdot \cos 2x \cdot \cos 3x$ .
12. Using Lagrange's multipliers method find the minimum value of  $u$ , where  $u = a^3x^2 + b^3y^2 + c^3z^2$  with the condition  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ .
13. Prove that the radius of curvature at any point of the cycloid  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$  is  $4a \cos \frac{\theta}{2}$ .
14. Solve the equation  $81x^3 - 18x^2 - 36x + 8 = 0$  whose roots are in harmonic progression.
15. Express  $\frac{\sin 6\theta}{\sin \theta}$  in terms of  $\cos \theta$ .
16. Show that the eccentric angles at the extremities of a pair of semi conjugate diameters of an ellipse differ by a right angle.

17. Derive the polar equation  $\frac{l}{r} = 1 + e \cos \theta$  of a conic.

18. Show that in a conic the semi-latus rectum is the harmonic mean between the segments of a focal chord.

**PART – C**

**Answer any TWO question**

**(2 x 20 = 40)**

19. If  $y = \sin(m \sin^{-1} x)$ , prove that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$  and  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ .

20. (a) Find the angle of intersection of the curves  $r = a(1 + \cos \theta)$  and  $r = b(1 - \cos \theta)$ .

(b) Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

21. Solve the equation  $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$ .

22. (a) Sum to infinity  $c \sin \alpha - \frac{c^2}{2} \sin 2\alpha + \frac{c^3}{3} \sin 3\alpha + \dots \infty$ .

(b) If  $e$  and  $e_1$  are two extremities of hyperbola and its conjugate, show that  $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$ .

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